JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 108, NO. C6, 3175, doi:10.1029/2001JC001222, 2003

# Parameterization of the Mueller matrix of oceanic waters

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Received 16 November 2001; revised 5 August 2002; accepted 5 March 2003; published 4 June 2003.

[1] This paper is devoted to the parameterization of the Mueller matrix of oceanic water measured by *Voss and Fry* [1984]. Their matrix was obtained as an average of many observations in different parts of the world ocean, including the Gulf of Mexico and the Pacific and Atlantic Oceans. The results presented here can be used as input data for the theoretical polarized radiative transfer studies in oceanic waters. *INDEX TERMS:* 4271 Oceanography: General: Physical and chemical properties of seawater; 4275 Oceanography: General: Remote sensing and electromagnetic processes (0689); 4552 Oceanography: Physical: Ocean optics; *KEYWORDS:* light polarization, light scattering, phase matrix

**Citation:** Kokhanovsky, A. A., Parameterization of the Mueller matrix of oceanic waters, *J. Geophys. Res.*, *108*(C6), 3175, doi:10.1029/2001JC001222, 2003.

## 1. Introduction

[2] Light propagation in oceans is studied in the framework of the radiative transfer theory [*Mobley*, 1994]. For this, one needs to know local optical characteristics of oceanic water such as absorption and differential volume scattering coefficients [*Shifrin*, 1988]. The main problem of the theoretical modeling lays in the fact that both concentration of suspended particles and their geometrical parameters such as size, shape, and structure are highly variable both in time and space domains.

[3] The situation becomes even more difficult if one is interested in the exploration of the light polarization in ocean water. This problem, for instance, is of importance due to a high correlation of the degree of polarization with the concentration of suspended particles [*Ivanoff*, 1975], which potentially has important practical applications. The model of spherical particles, which is used even in up-to-date studies in ocean optics [*Stramski et al.*, 2001], is totally inapplicable if one has a goal to describe the Mueller matrix [*Bohren and Huffman*, 1983; *Kokhanovsky*, 2001] of oceanic water. For instance, the element $P_{22}$ of this matrix is equal to $P_{11}$ for spherical polydispersions. This is not confirmed by measurements both in open ocean and coastal areas [*Kadyshevich et al.*, 1974].

[4] Clearly, the theory of light scattering by nonspherical particles should be used in this case. However, both the complexity of the nonspherical scattering theory and the high variability of ocean water microstructure make the solution of the problem highly complicated. With this in mind, we propose here the parameterization of the phase matrix of oceanic waters, based on experimental measurements. The experimental results used were obtained by *Voss and Fry* [1984], using an electro-optic light scattering polarimeter. Measurements were done on samples from the Atlantic and Pacific Oceans and Gulf of Mexico. Only averaged experimental data for the matrix, obtained in different locations and time intervals, are taken for the parameterization.

[5] The parameterization obtained can be used, for example, in the combination with analytical results of the polarized light transfer theory [see, e.g., *Zege and Chaikovskaya*, 1996, 1998, 1999].

## 2. Mueller Matrix

[6] The Mueller matrix relates Stokes vectors of incident and scattered light. It has the following general form [*Bohren and Huffman*, 1983]:

$$\hat{P} = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{pmatrix}, \tag{1}$$

where elements $P_{13}$, $P_{14}$, $P_{23}$, $P_{24}$, $P_{31}$, $P_{32}$, $P_{41}$, $P_{42}$, $P_{34}$, $P_{43}$ for oceanic water take very small values [*Voss and Fry*, 1984] and can be neglected for most studies. This, of course, simplifies theoretical analysis. However, these small values in the right upper and left lower $2 \times 2$ submatrices (and also elements $P_{34}$, $P_{43}$) should not be considered as exact zeroes in general. They could be of importance in the solution of selected ocean optics problems, especially focused on the chiroptical spectroscopy of oceanic waters [*Kokhanovsky*, 2003]. Also, there are experimental indications that these elements can have quite large values in single cases [*Kadyshevich et al.*, 1974]. This could be due to orientation, anisotropy, or chirality of particles. We, however, will neglect all these effects and consider the matrix of the form

$$\hat{P} = \begin{pmatrix} P_{11} & P_{12} & 0 & 0 \\ P_{21} & P_{22} & 0 & 0 \\ 0 & 0 & P_{33} & 0 \\ 0 & 0 & 0 & P_{44} \end{pmatrix}. \tag{2}$$

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0148-0227/03/2001JC001222$09.00

This form has been confirmed by experiments of *Voss and Fry* [1984]. They also found that $P_{12} \approx P_{21}$ and $P_{33} \approx P_{44}$. Then the normalized Mueller matrix $\hat{p} = \hat{P}/P_{11}$ takes the following form:

$$\hat{p} = \begin{pmatrix} 1 & p_{12} & 0 & 0 \\ p_{12} & p_{22} & 0 & 0 \\ 0 & 0 & p_{33} & 0 \\ 0 & 0 & 0 & p_{33} \end{pmatrix}. \tag{3}$$

We are interested in the modeling of the dependencies of elements $p_{12}$, $p_{22}$, and $p_{33}$ on the scattering angle θ. The dependence $P_{11}(\theta)$ was studied by many authors [see, e.g., *Petzold*, 1972; *Gordon*, 1974; *Shifrin*, 1988; *Stramski et al.*, 2001; *Mehrtens and Martin*, 2002] and therefore is not considered here. Note that the degree of polarization $p(\theta) \equiv -p_{12}(\theta)$ [*Kokhanovsky*, 2003].

[7] The elements of the normalized matrix $\hat{p}$ of oceanic water are given by simple analytical equations, if only molecular light scattering is accounted for (no particle scattering) [*Shifrin*, 1988],

$$p_{12}^{m}(\theta) = -\frac{p^m(90^\circ)\sin^2\theta}{1 + p^m(90^\circ)\cos^2\theta}, \tag{4}$$

$$p_{22}^{m}(\theta) = \frac{p^m(90^\circ)(1+\cos^2\theta)}{1 + p^m(90^\circ)\cos^2\theta}, \tag{5}$$

$$p_{33}^{m}(\theta) = \frac{2p^m(90^\circ)\cos\theta}{1 + p^m(90^\circ)\cos^2\theta}, \tag{6}$$

where $p^m(90^\circ) \equiv -p_{12}^m(90^\circ)$ and the subscript "$m$" underlines the fact that equations (4)–(6) describe molecular light scattering only. Note that $p^m(90^\circ) = 0.835$ at the wavelength equal to 436 nm and the temperature 20°C [*Shifrin*, 1988]. It is easy to show that the parameter $p^m(90^\circ)$ gives the degree of polarization (at $\theta = 90^\circ$) of initially unpolarized light, which is due to the scattering process by an idealized sample of oceanic water free from particles.

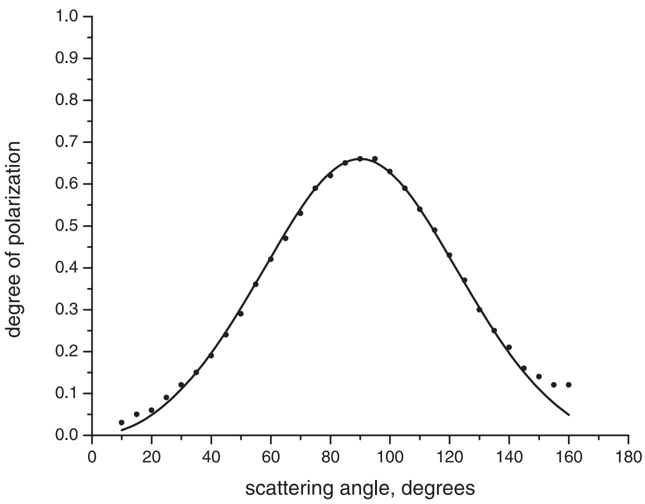


**Figure 1.** The dependence of the degree of polarization $p(\theta) = -p_{12}(\theta)$ on the scattering angle θ according to equation (7) (line) and measurements (dots).

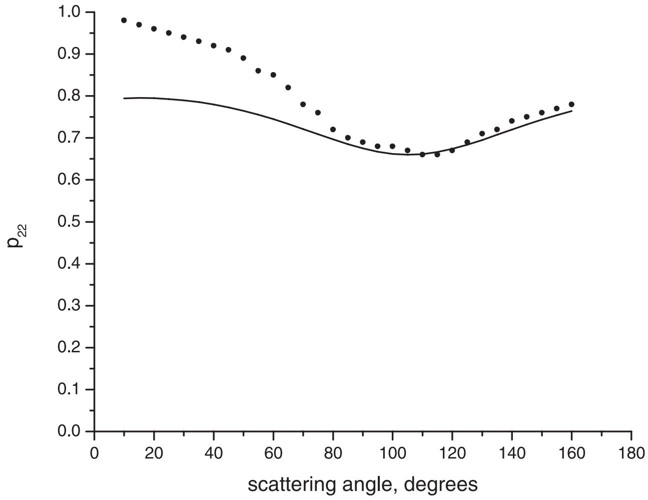


**Figure 2.** Same as in Figure 1 but for $p_{22}(\theta)$, given by equation (8).

[8] Generally speaking, the account for particle scattering modifies equations (4)–(6). Our task is to find analytical equations similar to equations (4)–(6), but for natural oceanic waters. For this we use experimental data obtained by *Voss and Fry* [1984]. We show experimental data by symbols in Figures 1–3. Lines correspond to following equations:

$$p(\theta) = \frac{p(90^\circ)\sin^2\theta}{1 + p(90^\circ)\cos^2\theta}, \tag{7}$$

$$p_{22}(\theta) = \frac{p(90^\circ)(1+\cos^2 z)}{1 + p(90^\circ)\cos^2 z}, \tag{8}$$

$$p_{33}(\theta) = \frac{2p(90^\circ)\cos\theta}{1 + p(90^\circ)\cos^2\theta}. \tag{9}$$

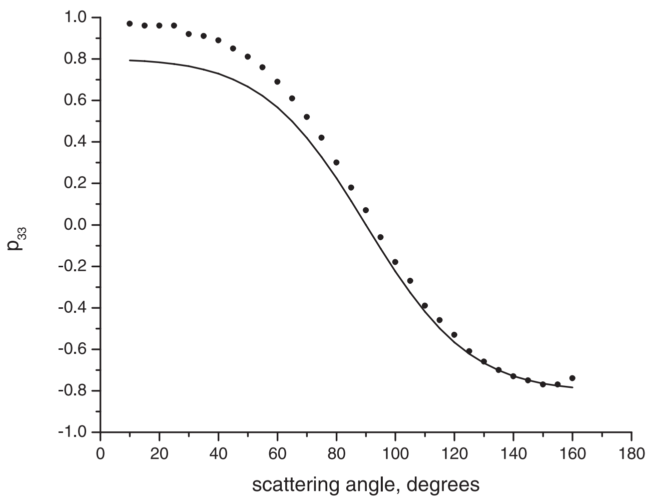


**Figure 3.** Same as in Figure 1 but for $p_{33}(\theta)$, given by equation (9).

[9] They differ from equations (4)–(6) only due to a different value of the degree of polarization at 90° ($p(90°)$ = 0.66), which was taken from *Voss and Fry* [1984, Table 2]. Also, the parameter $z = \theta - \theta_0$ was introduced to describe the shift of the minimum in the dependence $p_{22}(\theta)$ from the angle $\theta = 90°$(compare equations (5) and (8)). We found that $\theta_0 = 0.25$ for the data given by *Voss and Fry* [1984].

[10] First of all, note that equation (7) works well for natural oceanic waters. Thus it is enough to know the degree of polarization at 90° to characterize the degree of polarization in the range of scattering angles 20–150°. For smaller and larger angles, where the degree of polarization is below 15%, however, the discrepancy increases.

[11] Figures 2 and 3 show that equations (8) and (9) can be used also for modeling functions $p_{22}(\theta)$, $p_{33}(\theta)$ in the backward hemisphere. In the forward hemisphere, however, equations (8) and (9) contradict measurements. Therefore they should be modified. This necessity arises from the fact that equations (4)–(6) describe the dipole scattering mechanism only. In the forward scattering range, however, the contribution of multipole scattering (which is due to large oceanic particles) is not negligible [*Shifrin*, 1988].

[12] This contribution can be estimated, taking into account that $P_{22} \approx P_{33} \approx P_{11}$ (see results of measurements in Figures 2 and 3) for small angles. We approximate elements $P_{11}$, $P_{22}$, $P_{33}$ for multipole scattering at small angles by the exponential function $\xi\exp(-\alpha\theta)$ with free parameters $\alpha$ and $\xi$, which is a standard approximation as far as the analytical radiative transfer in oceanic water is concerned [*Zege et al.*, 1991]. Then it follows instead of equations (8) and (9),

$$p_{22}(\theta) = \frac{p(90°)(1+\cos^2(\theta-\theta_0)) + \xi\exp(-\alpha\theta)}{1+p(90°)\cos^2(\theta-\theta_0) + \xi\exp(-\alpha\theta)}, \tag{10}$$

$$p_{33}(\theta) = \frac{2p(90°)\cos\theta + \xi\exp(-\alpha\theta)}{1+p(90°)\cos^2\theta + \xi\exp(-\alpha\theta)}. \tag{11}$$

The first term in the nominator of equation (10) is due to the dipole scattering and the second one is due to the multipole scattering. The same applies to the nominator of equation (10). A similar interpretation can be given to dominators.

[13] Parameters $\alpha$ and $\xi$ in equations (10) and (11) were obtained by fitting procedure, using *Voss and Fry* [1984] data. The result is: $\alpha = 4$ rad$^{-1}$, $\xi = 25.6$. The comparison of equations (10) and (11) with experimental data at these values of $\alpha$, $\xi$ and $p(90°) = 0.66$, $\theta_0 = 0.25rad$ is given in Figures 4 and 5. We see that the correspondence of experimental data and fitting curves is excellent (with a minor exception in the range of angles 40–60° for $p_{22}$).

[14] This makes it possible to state that the angular dependence of the normalized Mueller matrix of oceanic water can be described in a good approximation, if only 4 parameters (e.g., $p(90°)$, $\theta_0$, $\alpha$ and $\xi$) are known. These parameters have a clear physical sense as it is shown above. Note that $\theta_0 = \alpha^{-1}$ in the case presented here. Alternatively, one can use the following set of parameters: $p(90°)$, $\theta_0$, $p_{22}(\theta_0)$, $p_{33}(\theta_0)$. This is a major conclusion of this work.

[15] The accuracy of proposed approximate equations (7), (10), and (11) is studied in Figure 6. One can see that the

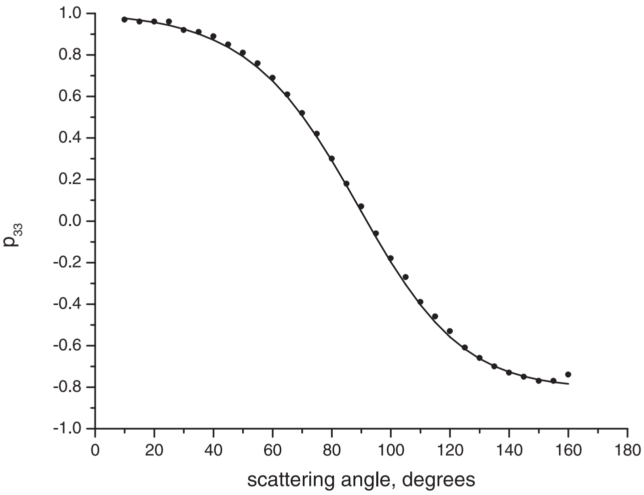


**Figure 5.** Same as in Figure 1 but for $p_{33}(\theta)$, given by equation (11).

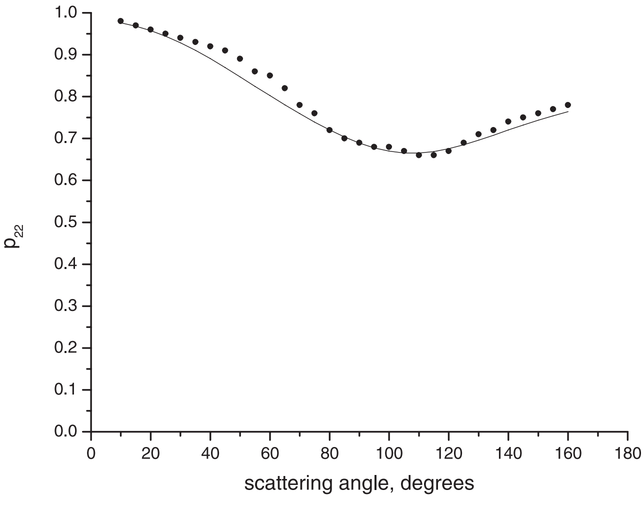


**Figure 4.** Same as in Figure 1 but for $p_{22}(\theta)$, given by equation (10).

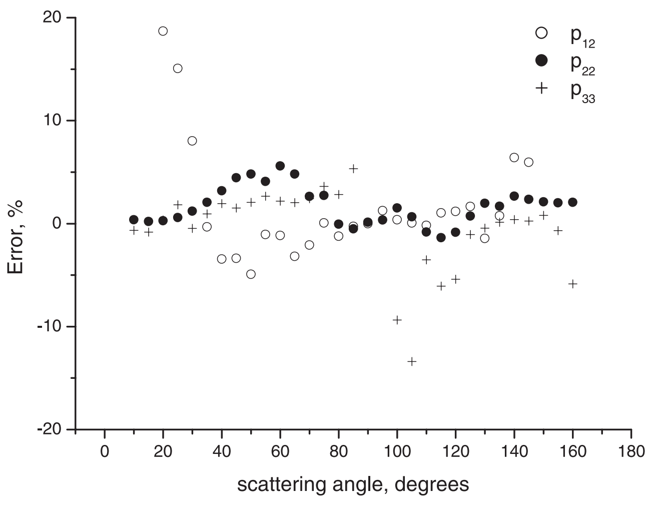


**Figure 6.** Relative errors of approximation (7), (10), and (11) for elements $p_{12}$, $p_{22}$, $p_{33}$.

error of the parameterization is typically in the range [−5%, 5%]. It increases for scattering angles, where normalized matrix elements approach zero. We can state that the relative errors of approximations (7), (10), and (11) are generally smaller or comparable to the errors of experiments involved. This confirms the fact that they capture correctly the behavior of functions $p_{12}(\theta)$, $p_{22}(\theta)$, and $p_{33}(\theta)$ in natural oceanic waters.

## 3. Conclusion

[16] In conclusion, we proposed here equations (7), (10), and (11) for the parameterization of the elements of the normalized Mueller matrix (see equation (3)) of oceanic water. They can be used for the theoretical modeling (both numerical and analytical) of the polarized radiative transfer in oceanic waters. Another important point is that they suggest convenient analytical forms for the description of experimental data. The detailed relation of four free parameters ($p(90°)$, $\theta_0$, $\alpha$, $\xi$) to the microstructure of oceanic waters should be clarified in future research.

[17] **Acknowledgments.** This work was supported by the ISTC Project B–519p. The author is grateful to L. I. Chaikovskaya and E. P. Zege for important discussions of the problem studied here.

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